# Simple Logistic Regression

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IBM SPSS Statistics Version 22 screenshots are copyrighted to IBM Corp.

#### Outlines

- Introduction
- Odds ratio vs relative risk
- Simple logistic regression
- Analysis in SPSS
  - 1. Descriptive Statistics
  - 2. Univariable Analysis
  - 3. Interpretation



- 1.Understand the concepts of odds and risk, and their relations with logistic regression
- 2.Perform simple logistic regression in SPSS
- 3.Identify and interpret the results

## Introduction

- Logistic regression is used when:
  - Dependent Variable, DV: A binary categorical variable
    [Yes/No], [Disease/No disease] i.e the outcome.
- Simple logistic regression Univariable:
  - Independent Variable, IV: A categorical/numerical variable.
- Linear Regression?
  - Dependent Variable, DV: ???

## Introduction

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- Linear Regression?
  - Dependent Variable, DV: Numerical

## Introduction

- Simple Linear Regression
  - -y = a + bx
- Simple Logistic Regression
  - $-\log(\text{odds}) = a + bx$
  - That's why it is called "logistic" regression
  - Allows us to obtain *odds ratio*

- Association analysis for cross-tabulation of a binary factor with a binary outcome can be expressed as odds ratio.
- <u>Odds</u> is a measure of chance of disease occurrence in a specified group,

$$Odds = \frac{n_{disease}}{n_{no \, disease}}$$

 <u>Odds ratio</u>, OR is the ratio between the odds of two groups; the group with the risk factor and the group without the risk factor,

$$Odds \, ratio \, , OR = \frac{Odds_{factor}}{Odds_{no \, factor}}$$

 Odds ratio is applicable to all observational study designs (cohort, cross-sectional and case-control) -- does not imply a cause-effect association, but only plain association.

- In epidemiology, the association between a risk factor and a disease is expressed in terms of risk and relative risk.
- <u>Risk</u> is a measure of chance of disease occurrence in a specific group,

$$Risk = \frac{n_{disease}}{n_{group}}$$

• Relative risk is the ration between the risk in the group with the factor and the risk in the group without the risk factor,

Relative risk, RR = 
$$\frac{Risk_{factor}}{Risk_{no factor}}$$

 Relative risk is only appropriate to calculate risk and relative risk for cohort studies, because the cause-effect relationship is well defined.

• Odds Ratio, OR

- Applicable to all observational studies.

- Relative Risk, RR
  - Only cohort study.
- OR ≈ RR for rare disease, useful to determine risk from a case-control study.

| Factor vs Disease | Lung CA | No Lung CA |  |
|-------------------|---------|------------|--|
| Smoker            | 24 [a]  | 76 [b]     |  |
| Non-smoker        | 13 [c]  | 87 [d]     |  |

- Odds(smoker) = a/b = 24/76 = 0.32
- Odds(non-smoker) = c/d = 13/87 = 0.15
- $OR(Odds_{smoker}/Odds_{non-smoker}) = 0.32/0.15 = 2.13$
- Shortcut, OR = ad/bc = (24x87)/(76x13) = 2.11

| Factor vs Disease | Lung CA | No Lung CA |  |
|-------------------|---------|------------|--|
| Smoker            | 24 [a]  | 76 [b]     |  |
| Non-smoker        | 13 [c]  | 87 [d]     |  |

- Risk(smoker) = Proportion CAD = a/(a+b) = 0.24
- Risk(non-smoker) = Proportion CAD c/(c+d) = 0.13
- RR(Risk<sub>smoker</sub>/Risk<sub>non-smoker</sub>) = 0.24/0.13 = 1.85 ≈ OR,
  2.11

#### **Simple Logistic Regression**

- Simple Logistic Regression
  - $-\log(\text{odds}) = a + bx$
  - That's why it is called "logistic" regression
  - Allows us to obtain *odds ratio*
- Odds ratio,

$$OR = exp(b)$$

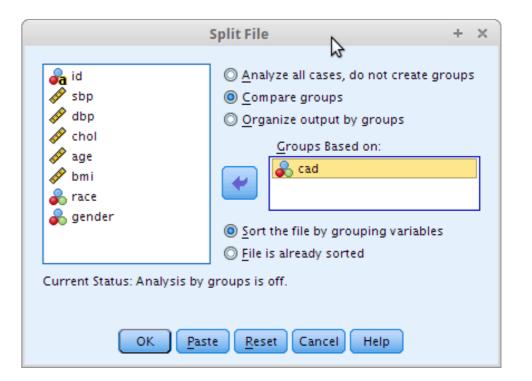
## **Analysis in SPSS**

- Dataset: slog.sav
- Sample size, n=200
- DV: *cad* (1: Yes, 0: No)
- IVs:
  - Numerical: *sbp* (systolic blood pressure), *dbp* (diastolic blood pressure), *chol* (serum cholesterol in mmol/L), *age* (age in years), *bmi* (Body Mass Index).
  - Categorical: *race* (0: Malay, 1: Chinese, 2: Indian), *gender* (0: Female, 1: Male)

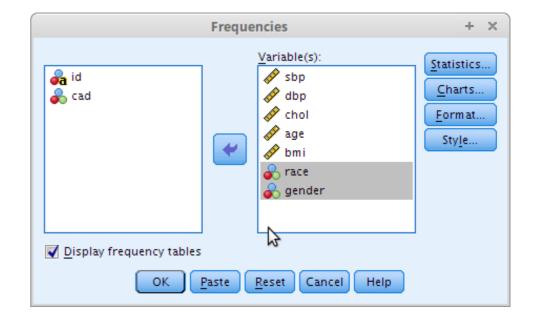
## **Steps in Multiple Logistic Regression**

- **1.Descriptive statistics**
- 2. Univariable analysis
- **3.Interpretation**

- Set outputs by CAD status.
  - Data → Split File →
    Select Compare groups
  - Set Groups Based on:
    cad, OK



- Obtain mean(SD) and n(%) by CAD group.
  - Analyze → Descriptive
    Statistics →
    Frequencies
  - Include relevant variables in Variables

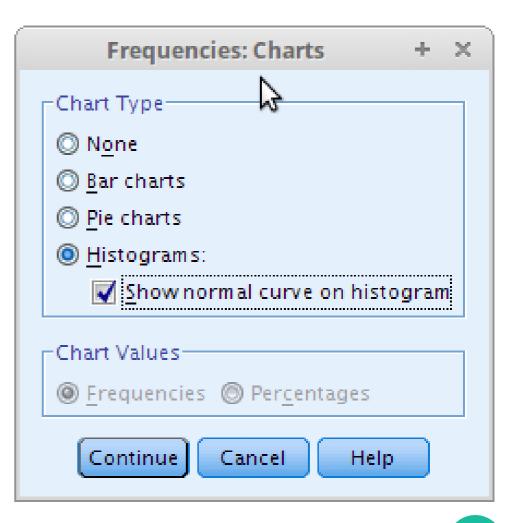


- Cont...
  - Statistics → tick →
    Continue

| Frequencies: Statist  | ics + ×   |
|---|---|
| Percentile Values       Quartiles      Cut points for: 10    equal groups      Percentile(s): | Kr<br>Central Tendency<br>Mean<br>Median<br>Mode<br>Sum |
|   | 📃 Va <u>l</u> ues are group midpoints                   |
| Dispersion  | Distribution  |
| 👿 Std. deviation 👿 Minimum  | Ske <u>w</u> ness                                       |
| 🔄 <u>V</u> ariance 👿 Ma <u>x</u> imum   | 📃 <u>K</u> urtosis                                      |
| 🔲 Ra <u>n</u> ge 📄 S. <u>E</u> . mean   |   |
| Continue  | Help  |

#### **Simple Logistic Regression**

- Cont...
  - Charts  $\rightarrow \underline{tick} \rightarrow$ Continue  $\rightarrow$  OK



#### • Results

|           | Statistics       |         |                                   |                                    |                                       |                     |                        |                |                  |  |
|-----------|------------------|---------|-----------------------------------|------------------------------------|---------------------------------------|---------------------|------------------------|----------------|------------------|--|
| cad coron | iary artery dise | ase     | sbp Systolic<br>Blood<br>Pressure | dbp Diastolic<br>Blood<br>Pressure | chol serum<br>cholesterol<br>(mmol/l) | age Age in<br>Years | bmi Body<br>Mass Index | race ethnicity | gender<br>gender |  |
| 0 no cad  | Ν                | Valid   | 163                               | 163                                | 163                                   | 163                 | 163                    | 163            | 163              |  |
|           |                  | Missing | 0                                 | 0                                  | 0                                     | 0                   | 0                      | 0              | 0                |  |
|           | Mean             |         | 129.29                            | 80.80                              | 6.0970                                | 45.15               | 36.9086                | .94            | .47              |  |
|           | Median           |         | 124.00                            | 80.00                              | 6.0500                                | 44.00               | 37.9000                | 1.00           | .00              |  |
|           | Std. Deviatio    | n       | 22.264                            | 12.607                             | 1.16633                               | 8.412               | 3.77178                | .826           | .500             |  |
|           | Minimum          |         | 88                                | 56                                 | 4.00                                  | 31                  | 25.30                  | 0              | 0                |  |
|           | Maximum          |         | 218                               | 120                                | 9.35                                  | 62                  | 41.20                  | 2              | 1                |  |
|           | Percentiles      | 25      | 114.00                            | 70.00                              | 5.3350                                | 37.00               | 36.1000                | .00            | .00              |  |
|           |                  | 50      | 124.00                            | 80.00                              | 6.0500                                | 44.00               | 37.9000                | 1.00           | .00              |  |
|           |                  | 75      | 140.00                            | 90.00                              | 6.7650                                | 52.00               | 39.2000                | 2.00           | 1.00             |  |
| 1 cad     | N                | Valid   | 37                                | 37                                 | 37                                    | 37                  | 37                     | 37             | 37               |  |
|           |                  | Missing | 0                                 | 0                                  | 0                                     | 0                   | 0                      | 0              | 0                |  |
|           | Mean             |         | 143.76                            | 88.97                              | 6.6459                                | 47.43               | 36.4464                | .97            | .65              |  |
|           | Median           |         | 138.00                            | 90.00                              | 6.6550                                | 50.00               | 37.1248                | 1.00           | 1.00             |  |
|           | Std. Deviatio    | n       | 25.611                            | 12.171                             | 1.17041                               | 8.796               | 3.99414                | .833           | .484             |  |
|           | Minimum          |         | 100                               | 70                                 | 4.13                                  | 33                  | 25.50                  | 0              | 0                |  |
|           | Maximum          |         | 224                               | 114                                | 9.05                                  | 61                  | 45.03                  | 2              | 1                |  |
|           | Percentiles      | 25      | 122.00                            | 78.00                              | 5.9537                                | 38.50               | 34.0802                | .00            | .00              |  |
|           |                  | 50      | 138.00                            | 90.00                              | 6.6550                                | 50.00               | 37.1248                | 1.00           | 1.00             |  |
|           |                  | 75      | 159.00                            | 97.00                              | 7.2875                                | 55.00               | 38.8146                | 2.00           | 1.00             |  |

#### • Results

| cad coronary artery disease |       | Frequency | Percent | Valid Percent | Cumulative<br>Percent |       |
|-----------------------------|-------|-----------|---------|---------------|-----------------------|-------|
| 0 no cad                    | Valid | 0 malay   | 60      | 36.8          | 36.8                  | 36.8  |
|                             |       | 1 chinese | 52      | 31.9          | 31.9                  | 68.7  |
|                             |       | 2 indian  | 51      | 31.3          | 31.3                  | 100.0 |
|                             |       | Total     | 163     | 100.0         | 100.0                 |       |
| 1 cad                       | Valid | 0 malay   | 13      | 35.1          | 35.1                  | 35.1  |
|                             |       | 1 chinese | 12      | 32.4          | 32.4                  | 67.6  |
|                             |       | 2 indian  | 12      | 32.4          | 32.4                  | 100.0 |
|                             |       | Total     | 37      | 100.0         | 100.0                 |       |

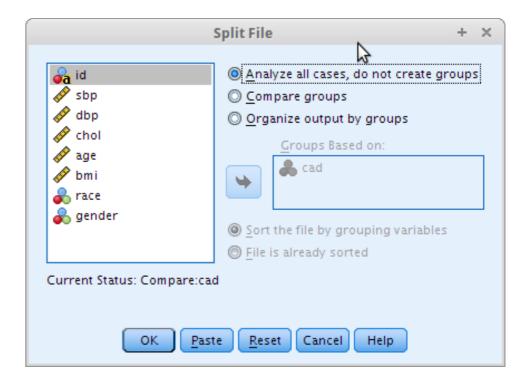
#### race ethnicity

#### gender gender

| cad corona | cad coronary artery disease |         | Frequency | Percent | Valid Percent | Cumulative<br>Percent |
|------------|-----------------------------|---------|-----------|---------|---------------|-----------------------|
| 0 no cad   | Valid                       | 0 woman | 87        | 53.4    | 53.4          | 53.4                  |
|            |                             | 1 man   | 76        | 46.6    | 46.6          | 100.0                 |
|            |                             | Total   | 163       | 100.0   | 100.0         |                       |
| 1 cad      | Valid                       | 0 woman | 13        | 35.1    | 35.1          | 35.1                  |
|            |                             | 1 man   | 24        | 64.9    | 64.9          | 100.0                 |
|            |                             | Total   | 37        | 100.0   | 100.0         |                       |

#### Results

- Look at histograms to decide data normality for numerical variables.
   Remember your Basic Stats!
- Caution! Reset back the data.
  - Data → Split File → Select
    Analyze all cases



– **OK** 

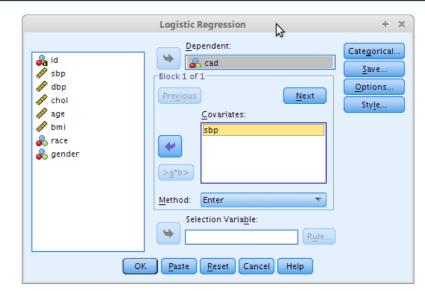
#### • Present the results in a table.

| Fac           | tors                       | CAD, <i>n</i> = 37<br>mean(SD)      | No CAD, <i>n</i> = 163<br>mean(SD)  |
|---------------|----------------------------|-------------------------------------|-------------------------------------|
| Systolic Blo  | od Pressure                | 143.8(25.61)                        | 129.3(22.26)                        |
| Diastolic Blo | ood Pressure               | 89.0(12.17)                         | 80.8(12.61)                         |
| Chole         | esterol                    | 6.6(1.17)                           | 6.1(1.17)                           |
| A             | ge                         | 47.4(8.80)                          | 45.2(8.41)                          |
| В             | MI                         | 36.4(3.99)                          | 36.9(3.77)                          |
| Race*         | Malay<br>Chinese<br>Indian | 13(35.1%)<br>12(32.4%)<br>12(32.4%) | 60(36.8%)<br>52(31.9%)<br>51(31.3%) |
| Gender*       | Male<br>Female             | 24(64.9%)<br>13(35.1%)              | 76(46.6%)<br>87(53.4%)              |

\*n (%)

- Perform Simple Logistic Regression on each IV
- Pay attention to whether IV is <u>numerical</u> or <u>categorical</u>

- Analyze <u>numerical</u> variables:
  - Analyze → Regression →
    Binary Logistic
  - Dependent: cad,
    Covariates: sbp
  - Click Options → Tick
    Iteration history, CI for
    exp(B) → Continue → OK
  - Repeat for *dbp*, *chol*, *age*, *bmi*



| Logistic Regression: Options +   |  |           |  |  |  |  |  |
|--|--|-----------|--|--|--|--|--|
| Statistics and Plots   |  |           |  |  |  |  |  |
| Classification plots   | Correlations of estimates                                      |           |  |  |  |  |  |
| 📗 <u>H</u> osmer-Lemeshow goodness-of-fit  | tteration history  |           |  |  |  |  |  |
| Casewise listing of residuals  | ✓ CI for exp(B): 95 %  |           |  |  |  |  |  |
| Outliers outside  Z std. dev.  All cases  Display  At <u>each step</u> © At <u>last step</u> | \$   |           |  |  |  |  |  |
| Probability for Stepwise<br>Entry: 0.05 Remo <u>v</u> al: 0.10                               | Classification c <u>u</u> toff:<br><u>M</u> aximum Iterations: | 0.5<br>20 |  |  |  |  |  |
| Conserve memory for complex analyse  | es or large <u>d</u> atasets                                   |           |  |  |  |  |  |
| ✓ Include constant in model  |  |           |  |  |  |  |  |
| Continue   | Cancel Help  |           |  |  |  |  |  |

#### Results

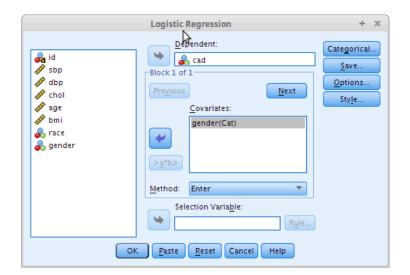
SBP *P*-value=0.001 by Wald test

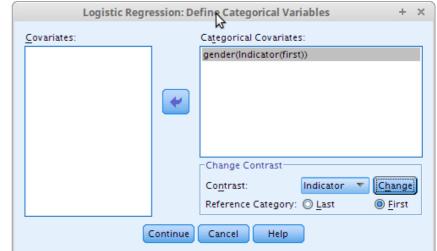
|         | Variables in the Equation |        |       |        |    |      |          |            |           |
|---------|---------------------------|--------|-------|--------|----|------|----------|------------|-----------|
|         |                           |        |       | /      |    |      |          | /95% C.I.f | or EXP(B) |
|         |                           | В      | S.E.  | Wald / | df | Sig. | Exp(B) / | Lower      | Upper     |
| Step 1ª | sbp                       | .024   | .007  | 10.290 | 1  | .001 | 1.024    | 1.009      | 1.039     |
|         | Constant                  | -4.684 | 1.039 | 20.303 | 1  | .000 | .009     |            |           |

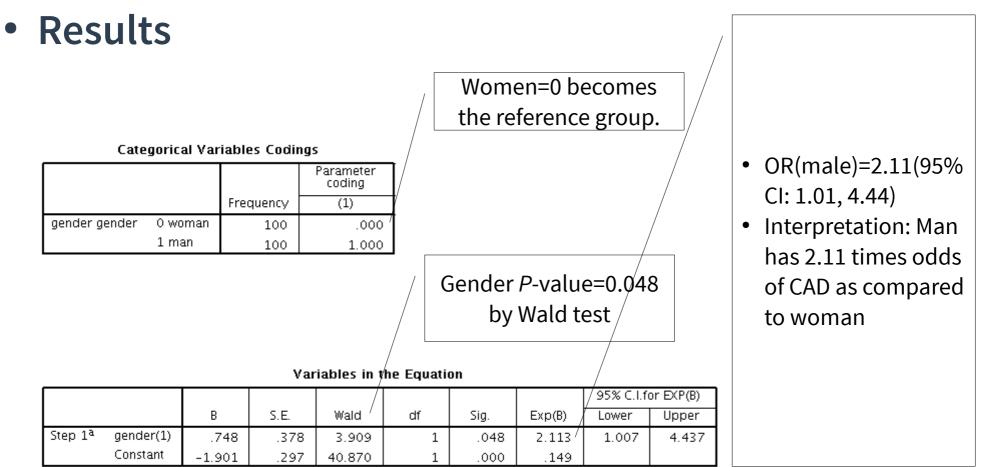
- Exp(B) is OR.
- OR(1 unit ↑ in SBP)
  =1.04(95% CI: 1.01,
  1.04)
- Interpretation: 1mmHg increase in SBP increase odds of CAD by 1.02 times

a. Variable(s) entered on step 1: sbp.

- Analyze <u>categorical</u> variables:
  - Dependent: cad,
    Covariates: gender
  - Click Categorical →
    Categorical Covariates: gender → Change
     Contrast → Reference
     Category: First →
     Change → Continue.
  - Repeat for *race*







a. Variable(s) entered on step 1: gender.

• OR values and P-values of IVs

Let's fill in the blanks

|                          | Factors                             |      | SE   | OR (95% CI)       | P-value |  |  |  |
|--------------------------|-------------------------------------|------|------|-------------------|---------|--|--|--|
| Systolic Blood Pressure  |                                     | 0.02 | 0.01 | 1.02 (1.01, 1.04) | 0.001   |  |  |  |
| Diastolic Blood Pressure |                                     |      |      |                   |         |  |  |  |
|                          | Cholesterol                         |      |      |                   |         |  |  |  |
|                          | Age                                 |      |      |                   |         |  |  |  |
|                          | ВМІ                                 |      |      |                   |         |  |  |  |
| Race                     | Chinese-vs-Malay<br>Indian-vs-Malay |      |      |                   |         |  |  |  |
| Gender                   | Man-vs-Woman                        | 0.75 | 0.38 | 2.11 (1.01, 4.44) | 0.048   |  |  |  |

#### 3. Interpretation

• Simple logistic regression of associated factors of coronary artery disease

| Fac  | b               | SE   | OR (95% CI)   | P-value           |       |  |
|--|-----------------|------|---|-------------------|-------|--|
| Systolic Blo   | ood Pressure    | 0.02 | 0.01  | 1.02 (1.01, 1.04) | 0.001 |  |
| Gender   | Man vs Woman    | 0.75 | 0.38  | 2.11 (1.01, 4.44) | 0.048 |  |
| 1mmHg increase<br>increase odds o<br>by 1.02 times, y<br>controlling for g                                   | of CAD<br>while |      | Man has 2.11 time<br>CAD as compared<br>while controlling | to woman,         |       |  |
| To obtain for 10mmHg increase in SBP<br>OR = $exp(c \times b) = exp(10 \times 0.05) = exp(0.5) = 1.22$ times |                 |      |   |                   |       |  |



#### Simple Logistic Regression